PEIRCE AND FISHER ON THE PLACE OF PROBABILITY IN ABDUCTIVE INFERENCE

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1. Introduction

In his analysis of inference into three types, deduction, induction, and abduction, C. S. Peirce maintains that probability plays an essential role in the first two, but not in the third. For a deductive argument, probability tells us the frequency with which the conclusion will hold given the premises; for an inductive argument, probability tells us the frequency with which the argument will hold true. However, probability has no role to play in abduction because there is, in Peirce's view, no common ground between the conclusion and the premises on which to base a calculation of frequencies. Nevertheless, there is something in the argument that makes the conclusion plausible to our mind. I will argue that this something is a probability which is, in many cases, quantifiable. This probability is the significance level of the statistical logic developed by R. A. Fisher in the 1920's.

2. Probability

In order to debate the role of probability in any type of argument, we must first have some agreement on the meaning of the term. In the context of this discussion, we need to examine how both Peirce and Fisher conceived of probability, and we need to show that these conceptions were essentially the same. Although it is well known that Peirce and Fisher held frequentist views of probability, there is a wide range of opinion in this camp. Indeed, Fisher once commented that he felt his views on statistical inference had more in common with those of the Bayesian Harold Jeffreys than with frequentists such as Jerzy Neyman and E. S. Pearson (BOX, 1978, p. 441).

In its mature form, Peirce's frequentist position incorporated a strong realist position with a slight variation on the standard statement of probability as the limit of frequencies. In a 1910 letter to Paul Carus, Peirce attributes errors in his earlier attempts to define probability to the nominalism of his youth and then notes,

> "I must show that the *will be's*, the actually *is's*, and the *have been's* are not the sum of the reals. They only cover actuality. There are besides *would be's* and *can be's* that are real." (PEIRCE, 1935-58, 8.216)

For Peirce, the frequency to which a probability refers is not an actual observed frequency, nor even one that could be observed. Although our knowledge of what "would be" can come only from observation, nevertheless, as Peirce says, "no collection of happenings can constitute one trillionth of one *per cent* of what might be." (PEIRCE, 1935-58, 6.328) This leads to the characterization of probability as a habit, the type of behavior expected from an individual under certain circumstances. In Peirce's words,

> "[T]o say that a die has a 'would-be' is to say that it has a property, quite analogous to any *habit* that a man might have. Only the 'would-be' of the die is presumably as such simpler and more definite than the man's habit as the die's homogenous composition and cubical shape is simpler than the nature of the man's nervous system and soul; and just as it would be necessary, in order to define a man's habit, to describe how it would lead him to behave and upon what sort of occasion —albeit this statement would by no means imply that the habit *consists* in that action —so to define the die's 'would-be', it is necessary to say how it would lead the die to behave on an occasion that would bring out the full consequence of the 'would-be'; and this statement will not of itself imply that the 'would-be' of the die *consists* in such behavior." (PEIRCE, 1935-58, 2.664)

Although many popular accounts confuse Fisher's frequentist views with those of J. Neyman and E. S. Pearson, Fisher was far more the realist. Where Neyman might prefer to speak of inductive behavior in place of inductive inference, Fisher clearly sought to use the tools of statistical inference to discover the state of nature. Analogous to Peirce's talk of "would be's", Fisher speaks of the population of possibilities and infinite hypothetical populations:

> "The salutary habit of repeating important experiments, or of carrying out original observations in replicate, shows a tacit appreciation of the fact that the object of our study is not the individual result, but the population of possibilities of which we do our best to make our experiments representative." (FISHER,

1991a, pp. 2-3)

> "The idea of an infinite hypothetical population is, I believe, implicit in all statements involving probability. If, in a Mendelian experiment, we say that the probability is one half that a mouse born of a certain mating shall be white, we must conceive of our mouse as one of an infinite population of mice which might have been produced by that mating. The population must be infinite for in sampling from a finite population the fact of one mouse being white would affect the probability of others being white, and this is not the hypothesis that we wish to consider; moreover, the probability may not always be a rational number. Being infinite the population is clearly hypothetical, for not only must the actual number produced by any parents be finite, but we might wish to consider the possibility that the probability should depend on the age of the parents, or their nutritional conditions. We can, however, imagine an unlimited number of mice produced upon the conditions of our experiment, that is, by similar parents, of the same age, in the same environment. The proportion of white mice in this imaginary population appears to be the actual meaning to be assigned to our statements of probability." (FISHER, 1971, vol. 2, p. 15)

Hence for Fisher, as for Peirce, a statement of probability refers to frequencies in a population, not of what is, or will be, or even what could be, but of what would be if the relevant procedure were to be reproduced indefinitely.

Fisher adds to this frequency account the perspective of the observer.

> "It has been often recognized that any probability statement, being a rigorous statement involving uncertainty, has less factual content than an assertion of certain fact would have, and at the same time has more factual content than a statement of complete ignorance. The *knowledge* required for such a statement refers to a well-defined aggregate, or population of possibilities within which the limiting frequency ratio must be exactly known. The necessary *ignorance* is specified by our inability to discriminate any of the different sub-aggregates having different frequency ratios, such as must always exist." (FISHER, 1991c, pp. 35-36)

For example, when I shake a die in my hand and toss it, I can discern certain facts which enable me to reliably state the frequency with which a six will occur, but I cannot discriminate to the point of identifying this particular throw as one of the subset of possible throws for which a six will definitely appear. If I could distinguish, say, the rolls which will yield an odd result from those that will yield an even result, this knowledge would focus my attention on specific subsets of the possible rolls, and my ignorance of further discriminations would lead me to assign

probabilities to these subsets in an appropriate manner.

We are thus led to analyze probability into three elements, a decomposition to which I believe both Peirce and Fisher would agree. In terms of Peirce's categories, we have a first, the habit of some mechanism, for example, a die along with a device for tossing it; a second, the frequencies in the population of possibilities, the "would be's" of the die; and a third, the discernment of the experimenter. It is the experimenter, in her mediation between mechanism and possibilities, who brings out the pragmatic meaning of probability in her development of the habit of induction.

3. Deduction, induction, and abduction

For Peirce the role of probability in deduction is straightforward: Given a deductive syllogism, probability tells us the frequency with which the conclusion will hold given the premises. For example, if we have a jar filled with red and green beads, 30% of which are known to be red, and we draw out a large handful, then, assuming the beads were well mixed, we may conclude that approximately 30% of the beads in our hand are red. More precisely, we could state a syllogism:

> 300 of the 1000 beads in this jar are red.
>
> This handful of 100 beads is from this jar.
>
> Hence between 25 and 35 of the beads in my hand are red.

The conclusion holds with a certain probability, namely, in the population of possible draws of 100 beads from this jar, we can calculate that approximately 79% of them will have between 25 and 35 red beads.

Exchanging the conclusion of a deductive syllogism with its major premise gives us an inductive syllogism. Continuing our example, if we have a handful of beads, 30% of which are red, drawn from this jar, then we may conclude that approximately 30% of the beads in the jar are red. In Peirce's view there is no way to attach a probability to the conclusion of this argument: To say that a certain percentage of the time approximately 30% of the beads in the jar

are red would mean that we are sampling from some possible population of jars, some of which have 30% red beads and some of which do not. But to Peirce, we are talking here about discovering the true state of nature and not of sampling from some population of possible universes. For example, Peirce says,

> "[I]t may be conceived, and often is conceived, that induction lends a probability to its conclusion. Now that is not the way in which induction leads to the truth. It lends no definite probability to its conclusion. It is nonsense to talk of the probability of a law, as if we could pick universes out of a grab-bag and find in what proportion of them the law held good." (PEIRCE, 1935-58, 2.780)

As Ian Hacking has pointed out, Peirce understood what Neyman and Pearson later made explicitly quantifiable: The probability in an inductive inference attaches to the argument as a whole, not to the conclusion. (HACKING,1980 and 1990) The conclusion of an inductive syllogism is, by itself, either true or false, independent of the particular sample we have observed. However, using proper sampling procedures, we can say what percentage of the possible samples will yield true conclusions. For example, consider this form of our inductive syllogism:

> 30 of the 100 beads in this handful are red.
>
> This handful of 100 beads is from this jar of 1000 beads.
>
> Hence between 21 and 39 percent of the beads in this jar are red.

Here the numbers 21 and 39 have been computed with a procedure known to lead to a correct conclusion 95% of the time. Given another sample, the procedure will yield a different range of values. Hence the probability attaches to the argument as a whole, not to the particular interval computed.

Fisher would not agree entirely with the above analysis. Although he forcefully rejected the Bayesian arguments that are typically used to assign a probability to the conclusion of an inductive inference, he tried throughout much of his life to replace the Bayesian argument with his own fiducial argument. On the one hand he wished to rid statistical analysis of Bayesian prior distributions which had no grounding in observation, yet at the same time he so disliked the talk of inductive behavior that came from the Neyman-Pearson school of statistics that he was deter-

mined to preserve the stronger conclusions of the Bayesian arguments. Hence Fisher, like Peirce before him, was able to reveal the weakness of the Bayesian argument from prior distributions based on ignorance, yet he did not see the deeper argument of Peirce: Not only does the Bayesian approach fail, but any approach that attempts to attach a probability to the conclusion of an inductive argument must fail as long as one holds to a frequency based approach to probability.

Exchanging the conclusion of a deductive syllogism with its minor premise yields what Peirce calls an abductive syllogism. For our example, if some jars of red and green beads have 30% red beads and we have a sample, drawn from some jar, which contains 30% red beads, then we may infer that our sample is from one of those containing 30% red beads. Although Peirce would say that the conclusion of this inference is plausible, he would also contend that its plausibility cannot be quantified as a probability; as in the case of induction, the conclusion is either true or false. Nor can we say that the argument holds with a certain frequency; not being given in the premises the jar from which the sample came, there is no ground for the calculation of a probability. In this view the sample is not drawn by some random mechanism, sometimes from this jar and sometimes from another; rather, the sample provides us with information about some state of nature and it is up to us to classify it to the best of our ability.

For the most part Peirce is adamant in this position. For example, in a letter written in 1911, he contends that Aristotle's apparent neglect of abduction as a form of inference

> "... needs no explanation if we suppose Aristotle was looking at Induction from the point of view of the doctrine of chances. Because then he would find that he must keep the minor premiss as a premiss, since otherwise he could not make any application of the doctrine of chances not having two premisses both speaking of [an] accountable number of objects." (PEIRCE, 1976, vol. 3, p. 200)

Yet in a letter to Langley in 1901 Peirce indicates a source for the plausibility that the conclusion of an abduction commands. He writes,

> "A phenomenon is observed having something peculiar about it. Rumination leads me to see that *if* a certain state of things existed, of whose actual existence I know nothing, that phenomenon would certainly occur, or, at any rate, would in all probability occur." (BRENT, 1993, p. 276)

That is, the conclusion of an abductive argument is plausible if the minor premise is highly

probable given the major premise and the conclusion. This is an indication of how probability plays a role in abduction.

For our example, if the sample of beads really did come from a jar with 30% red beads, the observation of the minor premise, that is, that we have a sample with 30% red beads, is reasonably probable and hence plausible. However, using this criterion, other conclusions are plausible as well. For example, a conclusion stating that the sample is from a jar containing 25% red beads would also be plausible, since the probability of obtaining a sample with 30% red beads from such a jar is not inordinately small. More precisely, if 25% of the beads in a jar containing 1000 beads are red, then the probability that a sample of 100 beads from the jar would have 30% or more red beads is 0.1372, a little more than the probability of obtaining three heads in three tosses of a fair coin. But a conclusion stating that the sample is from a jar containing 20% red beads would not be plausible, there being a probability of only 0.0079, about the chance of obtaining seven heads in seven tosses of a fair coin, that a jar with 20% red beads would yield a sample with at least 30% red beads. Hence the argument is stronger for concluding that the sample *is not* from a jar with 20% red beads than it is for saying that it *is* from a jar with 30% red beads. In fact, even if these are the only two possible states of fact, the strength of the conclusion that the sample is from the jar with 30% red beads rests more on the implausibility of the other choice than on the plausibility of the conclusion itself.

The above line of reasoning is exemplified in the following example from Peirce.

> "I once landed at a seaport in a Turkish province; and as I was walking up to the house which I was to visit, I met a man upon horseback, surrounded by four horsemen holding a canopy over his head. As the governor of the province was the only personage I could think of who would be so greatly honored, I inferred that this was he." (PEIRCE, 1992, p. 189)

The key to this argument is not so much that if this man is the governor, then he would be honored in this way, but that if he were not the governor, then the probability that he would be seen traveling in this style would be extremely small, if not zero. Hence it is the probability of the minor premise given the major premise and the negation of the conclusion that makes the

conclusion of an abduction plausible. This is the probability that Fisher called the significance level.

In a 1926 paper, a precursor to his book *The Design of Experiments*, Fisher discusses the logic behind a typical agricultural experiment. An experimenter, looking for evidence that a certain fertilizer increases the yield of a certain crop, sows seed in two one acre plots, treats both plots alike except for the treatment of the fertilizer on one, and observes that the treated plot yielded a crop 10% larger than the other plot. The experimenter then forms the inference:

> Some fertilizers increase the yield of this crop.
>
> This fertilizer increased yield by 10%.
>
> Hence this fertilizer is one of those that increases yield.

Fisher realizes that this argument is not strong, and states that, in response to criticism,

> "[t]he early experimenter would have had to reply merely that he had chosen the land fairly, that he had no reason to expect one acre to be better than the other, and (possibly) that he had weighed the produce from these two acres in previous years and had never known them to differ by 10 per cent. The last argument alone carries any weight." (FISHER, 1971, vol. 2, p. 85)

Only the reply that a deviation this large has never been observed in prior years carries any weight because only it allows an estimation of the probability of the minor premise given the negation of the conclusion. If, for example, 20 years of past experience had never yielded so large a deviation, then we would have an observation which, if the fertilizer were such that it did not increase crop yield, would not occur more than once in twenty trials. In that case, the conclusion that this fertilizer is not one of those that increases yield would have a significance level less than 0.05, from which we could conclude that it is implausible and that the conclusion that this fertilizer does increase yield is plausible.

Unlike Peirce, Fisher does not divide nondeductive logic into two categories, induction and abduction, but refers to the whole of nondeductive logic as inductive inference. However, he draws an important distinction between what he calls the acceptance procedures of the Neyman-Pearson school of statistics and his own conception of tests of significance. An acceptance proce-

dure is a rule for making decisions: A company receives a large shipment from one of its suppliers; it samples the shipment; it then accepts or rejects the shipment according to a rule designed to minimize the probability of certain types of errors, such as rejecting a shipment when it actually contains an acceptably low percentage of defective items. On the other hand, a test of significance is an attempt to use observations from the world to classify a state of nature. Investigators use these tests to determine the genuineness of a genetic linkage or the reality of the response to a fertilizer treatment. As Fisher says,

> "The conclusions drawn from such tests constitute the steps by which the research worker gains a better understanding of his experimental material, and of the problems it presents." (FISHER, 1991c, p. 79)

For Fisher, there is deep gulf between these two types of inductive inference. As he says,

> "[O]ne of the deepest dissimilarities lies in the population, or reference set, available for making statements of probability. ... [W]here acceptance procedures are appropriate, the population lots of one or more items, which could be chosen for examination, is unequivocally defined. ... Whereas, the only populations that can be referred to in a test of significance have no objective reality, being exclusively the product of the statistician's imagination through the hypotheses which he has decided to test." (FISHER, 1991c, p. 81)

The difference lies in that acceptance procedures are inductive arguments, taking samples from fixed populations and making decisions based on arguments whose frequency of correctness can be calculated. In contrast, a significance test is an abductive argument, taking a set of observations from no specified population and attempting in the conclusion to classify them in the manner which is most reasonable.

This last quote from Fisher parallels an earlier quote from Peirce, namely, where he said probability does not enter into abduction, "not having two premises both speaking of [an] accountable number of objects." (PEIRCE, 1976, vol. 3, p. 200) Yet Fisher found a way to bring probability into the argument, showing that it is in measuring, with probability, the degree to which our observations agree with our hypothesis that we measure the plausibility of an inference.

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